



QUADRUPOLE FIELD ROTATION AND DISTORTION

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I. THE VERTICAL CENTER MEASUREMENTS

We have been analysing a group of data about measurement of the vertical center of the quadrupoles in the Main Ring.<sup>1,2</sup>

A typical example of data referring to one magnet is that shown below:

7060	A1061						
0.0490	0.0496	0.0481	0.0480	0.0472	0.0469	0.0442	0.0436
0.0511	0.0489	0.0491	0.0482	0.0478	0.0466	0.0436	0.0417
0.0572	0.0484	0.0479	0.0471	0.0468	0.0463	0.0447	0.0440

We have in the first two the identification number of the magnet and the location in the ring. The other 24 numbers are the vertical centers in inches, taken at different currents and from a "reference horizontal plane". The three rows of data correspond to +2", 0", and -2" respectively. A column corresponds to a current value. The eight (8) values of the current are as follows (in Amperes).

100	1000	2000	3000	4000	5000	6000	6500
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An example of how the centers are located is shown in Figure 1. The beam is supposed to move in the direction looking at the figure.

The measurements were made by summing the two integrated signals from two vertical loops, placed symmetrically around the



"reference horizontal plane", while the magnet was excited by a linear raising current up to 7000 A and having a slope of 2000 A/s.

We collected and analysed the data of 180 quadrupoles, all of them still sitting, at the date of writing, in the main ring tunnel. Thirty-eight quadrupoles are 4' long and their data have been linearly scaled to the length (7') of the other magnets in order to make all the data comparable.

For every magnet we calculated the following angles:

- $\alpha_{\text{out}}$  , the angle between the segment  $A_1A_2$  and the reference horizontal plane,
- $\alpha_{\text{in}}$  , the angle between the segment  $A_2A_3$  and the reference plane,
- $\theta$  , the angle between the segment  $A_1A_3$  and the reference horizontal plane,
- $\phi$  , the smallest of the two angles formed by the segments  $A_1A_2$  and  $A_2A_3$ .

All the angles are defined positive in the anti-clockwise direction as looking at Figure 1. The two angles  $\alpha_{\text{out}}$  and  $\alpha_{\text{in}}$  give information about the rotation of the magnetic field respectively of the outer and inner half-section of the quadrupole. The beam might behave differently accordingly whether it is travelling in the outside region ( $x > 0$ ) or in the inside region ( $x < 0$ ). In the case the beam is sitting at the center of the quadrupoles or continuously shifting from one side to another of the magnets, it might be more useful to consider the other two angles  $\theta$  and  $\phi$ . The former is the average rotation of the magnetic field in the

quadrupole, and the second a measure of the curvature (kink) of the quadrupole field. If  $\phi = 0$  there is no curvature; the kink is upward (i.e. the point  $A_2$  is above the segment  $A_1A_3$ ) when  $\phi > 0$ .

The results of our statistical computation are shown in Figures 2, 3, 4, and 5. The central line is the average of the distribution, versus current; the two external lines show the full rms deviation of the distribution around the average curve.

In order to make an idea of the reliability of the data displayed in these figures, let us give a look to the experimental errors:

1. All the measurements of the vertical center of one quadrupole (versus current and versus position) are affected by the same systematic error because of the accuracy of the vertical elevation of the "reference horizontal plane". The change of this error from one magnet to another is typically  $\pm 0.010''$ .

Since the angles defined above are all obtained as differences of the vertical centers, they are not affected by this kind of error.

2. An error was introduced in setting the orientation of the "reference horizontal plane". Typical value of this error is  $(\pm) 2$  or  $3$  mrad.

3. The probe sensitivity versus current is constant, at least in the range 1000 A to 7000 A. This introduces only a small error ( $\sim \pm 0.001''$ ) when the vertical centers at different currents are compared. At 100 A the sensitivity of the probe is reduced and the error is larger ( $\sim \pm 0.004''$ ).

## II. INTERPRETATION OF THE DATA

We shall limit our analysis to the angles  $\theta$  and  $\phi$  since the other two angles,  $\alpha_{out}$  and  $\alpha_{in}$ , are correlated to these.

## a. Quadrupole field rotation.

This is measured by  $\theta$ . Inspecting Figure 4, we see the average of the distribution is comparable to the statistical experimental error per magnet; probably it is the result of a systematic error introduced in the measurements. Since the standard deviations sum quadratically, subtracting the experimental error from the rms width of the distribution we obtain that:

the field in the quadrupoles is rotated with respect to the geometrical axis with the average within the statistical experimental error and an rms angle of about 5 mrad. This angle does not change with the current.

The rotation is known to cause the largest contribution to the coupling between the horizontal and the vertical motion. Its effect has already been estimated in another paper.<sup>3</sup>

Air-core skew quadrupoles are being installed in the main ring to overcome this effect.

## b. Curvature of the horizontal symmetry plane (kink).

This is measured by  $\phi$ . This angle is not affected by the experimental error on the orientation of the "reference horizontal plane", and its indirect measurement is rather accurate. By inspecting Figure 5, we observe that, at least up to the energy of 200 GeV, the average value is considerably smaller than the rms width, as we can see also from the next table.

Energy	$\phi_{ave}$	$\phi_{rms}$	error
inject	-3 mrad	25 mrad	$\pm 8$ mrad
200 GeV	-1 mrad	5 mrad	$\pm 2$ mrad

Thus in the following we shall assume the distribution has zero average.

A possible explanation of the kink is that the upper and lower sections of a quadrupole have different H-B curves and, hence, different remanent fields. This causes the vertical center at  $x = 0$ " to be displaced by a larger amount than that for the centers at  $x = \pm 2$ ", because of the different gap length at the three positions.

A kink can be regarded as a sextupole field, but rotated by  $30^\circ$  with respect to the standard configuration.

The equations for  $B_x$  and  $B_y$  are now (disregarding any constant term):

$$B_x = B'y + \frac{\phi}{2\ell} B' (x^2 - y^2)$$

$$B_y = B'x - \frac{\phi}{\ell} B' xy$$

where  $\ell = 2$  inches, and  $\phi$  is the kink angle of the particular magnet.  $B'$  is the gradient.

The equation of motion of a particle through one quadrupole, now writes

$$\frac{d^2 x}{ds^2} + Kx - K \frac{\phi}{\ell} yx = 0$$

$$\frac{d^2 y}{ds^2} - Ky - K \frac{\phi}{2\ell} (x^2 - y^2) = 0$$

$$K = \frac{B'}{B\rho_0} = \text{magnetic rigidity of the particle}$$

It is impossible to find an analytical solution of Eq. (1). Nevertheless we can approximately solve the system in two particular cases.

### III. VERTICAL CLOSED ORBIT DISTORTION

Let us neglect the  $y^2$  term at the last term of the second of Eq. (1) and replacy  $x$  by

$$X_p \frac{\delta p}{p}$$

where  $X_p$  is the momentum compaction length and  $\delta p/p$  the relative momentum deviation from the  $x = 0$  reference orbit. We have the equation

$$y'' - Ky = K \frac{\phi}{2\ell} X_p^2 \left(\frac{\delta p}{p}\right)^2$$

The inhomogeneous solution is a vertical closed orbit distortion. The rms value of the expected distortion is

$$\begin{aligned} \langle y \rangle &= \frac{\sqrt{M}}{2} L \bar{\beta} \frac{K \langle \Delta \rangle}{\sin \pi \nu} \\ \langle \Delta \rangle &= \frac{\phi_{\text{rms}}}{2\ell} \bar{X}_p^2 \left(\frac{\delta p}{p}\right)^2 \end{aligned}$$

where  $M$  is the number of quadrupoles,  $L$  the length of each quadrupole, and  $\bar{\beta}$ ,  $\bar{X}_p$  the average of  $\beta$  (second Twiss parameter) and  $X_p$ .

For the Main Ring we get ( $\sin \pi \nu \approx 1$ ):

at injection

$$\langle y \rangle = 43 \left(\frac{\delta p}{p}\right)^2 \text{ m,}$$

at higher energy

$$\langle y \rangle = 8 \left(\frac{\delta p}{p}\right)^2 \text{ m.}$$

Thus there is a large closed orbit distortion when the beam is moved away from the central ( $x = 0$ ) horizontal orbit. For  $\delta p/p = 10^{-2}$ , which corresponds to a few centimeter from the

$x = 0$  orbit, the rms vertical orbit is at least 4.3 mm (for  $\sin\pi\nu \sim 1$ ) at low energy, and at least 0.8 mm at higher energy.

Now we can replace the  $y$  by  $\langle y \rangle$  in the first of Eq. (1) and calculate the rms value of the  $\nu$ -shift caused by the distortion of the vertical closed orbit. We obtain

$$\Delta\nu = \frac{1}{4\pi} \sqrt{M} \bar{\beta} L K \frac{\phi}{\ell} \langle y \rangle$$

which gives, at the injection

$$\Delta\nu = 59 \left( \frac{\delta p}{p} \right)^2, \quad (\sin\pi\nu \sim 1)$$

and at higher energy

$$\Delta\nu = 12 \left( \frac{\delta p}{p} \right)^2, \quad (\sin\pi\nu \sim 1).$$

These numbers are small even for large  $\delta p/p$  ( $\sim 10^{-2}$ ). Also, the stopband width at either integer or half-integer resonance is small (about twice  $\Delta\nu$ ).

#### IV. MOMENTUM DEPENDENT COUPLING

We replace  $x$  by  $x_p \frac{\delta p}{p}$  to the third term in the first of the Eq. (1). We neglect the  $y^2$  term and we replace  $x^2$  by  $2x x_p \frac{\delta p}{p}$  in the second of the Eq. (1). We obtain

$$\frac{d^2 x}{ds^2} + Kx - \left( K \frac{\phi}{\ell} x_p \frac{\delta p}{p} \right) y = 0$$

$$\frac{d^2 y}{ds^2} - Ky - \left( K \frac{\phi}{\ell} x_p \frac{\delta p}{p} \right) x = 0$$

In the smooth approximation and taking only the zeroth harmonic of the coupling factor, we have

$$x'' + \nu_x^2 x = Cy$$

$$y'' + \nu_y^2 y = Cx$$

where prime is the derivative with respect to the longitudinal

phase angle, and

$$C = \sqrt{M} \frac{\phi_{\text{rms}}}{\ell} \bar{X}_p \frac{\delta p}{p}$$

For the NAL main ring it is, at the injection

$$C = 23 \frac{\delta p}{p}$$

and at higher energy

$$C = 4.6 \frac{\delta p}{p}$$

Comparing these numbers with the results of another paper<sup>3</sup> we infer their contributions to the horizontal-vertical coupling are rather small even for  $\delta p/p = 10^{-2}$ .



REFERENCES

1. C. Schmidt provided the information about the measurements procedure. (1972)
2. The data are part of the contents of the master magnetic tape prepared by John Schivell. For more information about this tape read J. Schiveel's internal report TM-366 (NAL, April, 1972).
3. L. C. Teng, TM-382 (NAL, July, 1972).

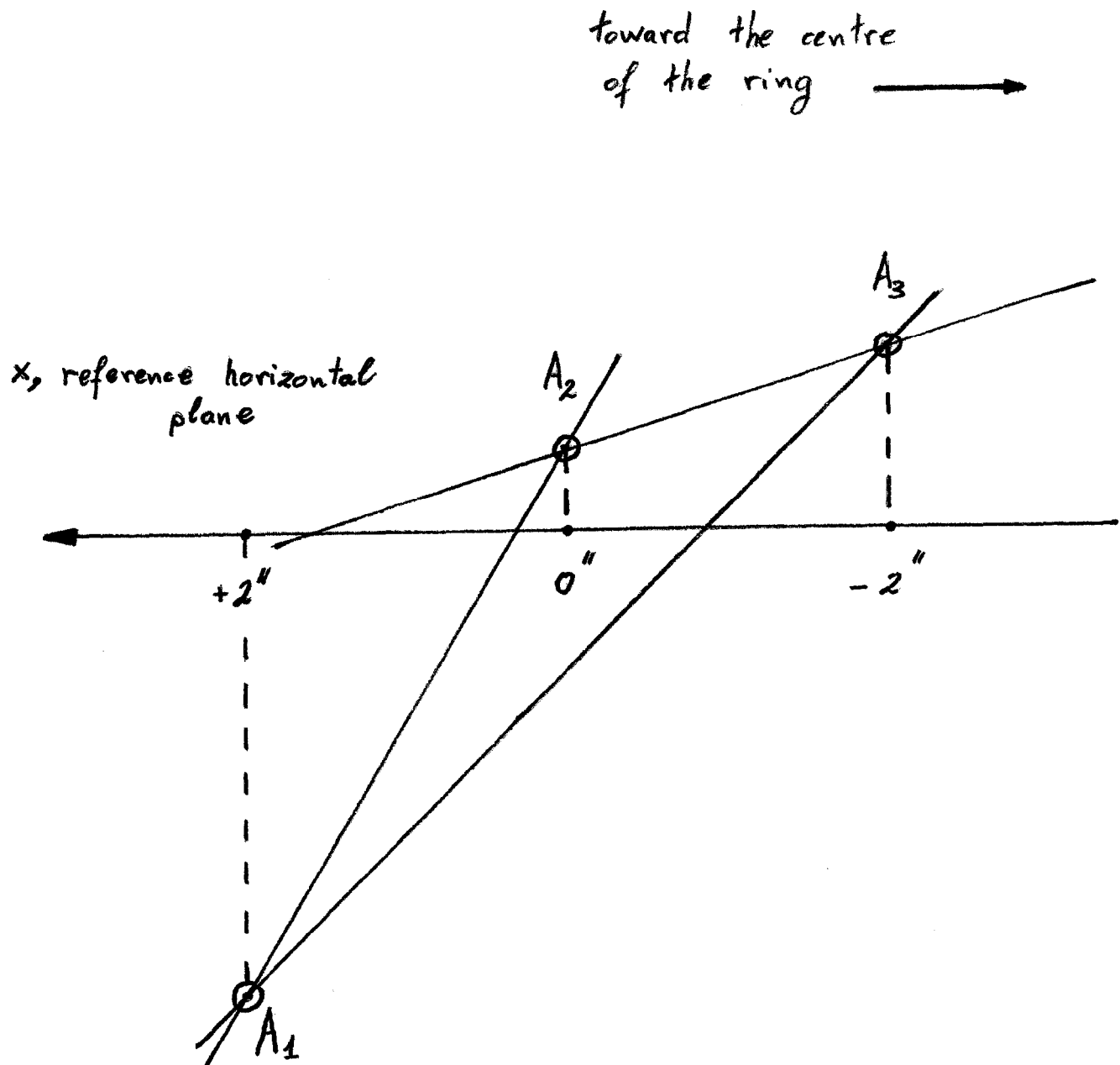
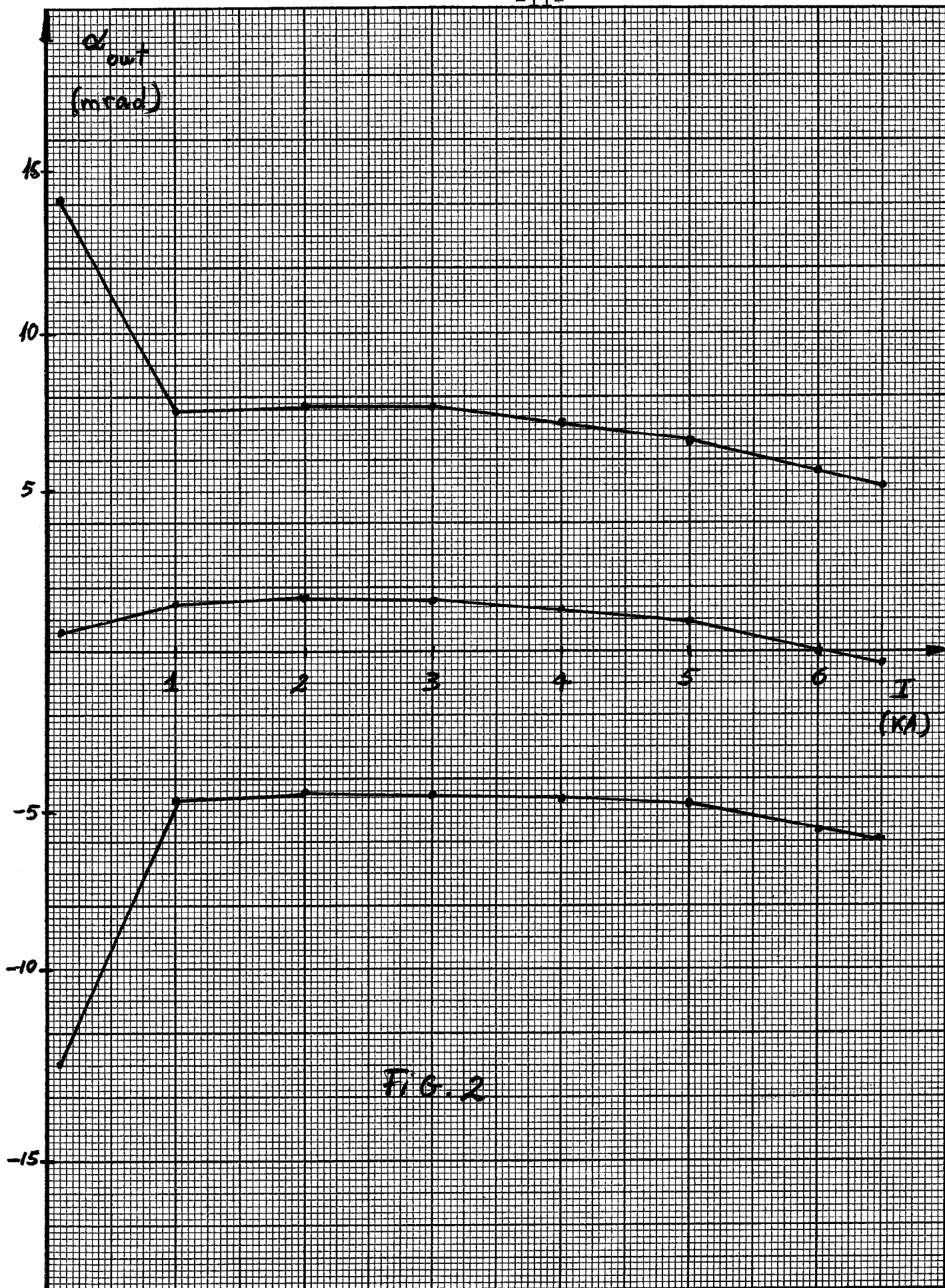
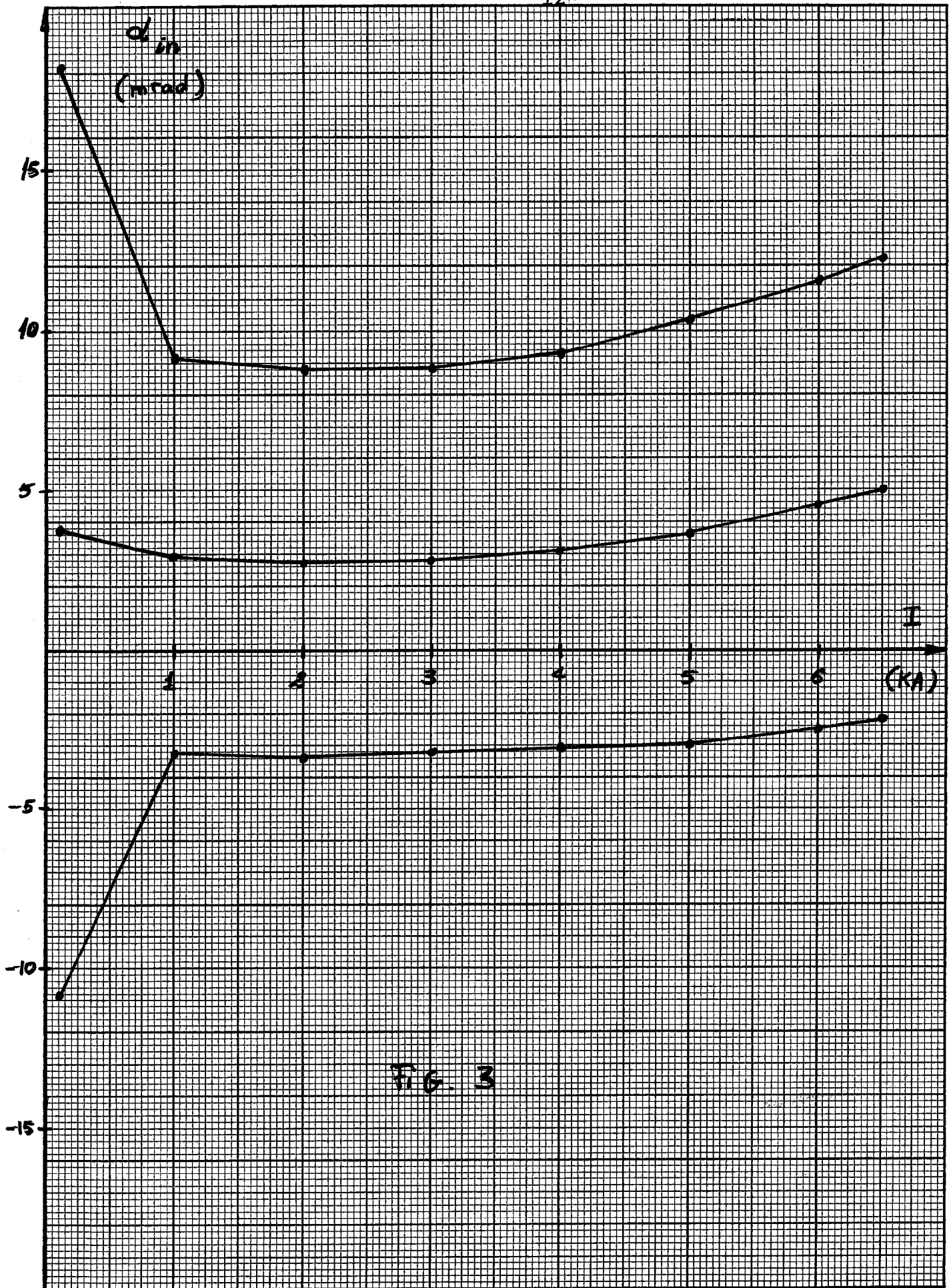
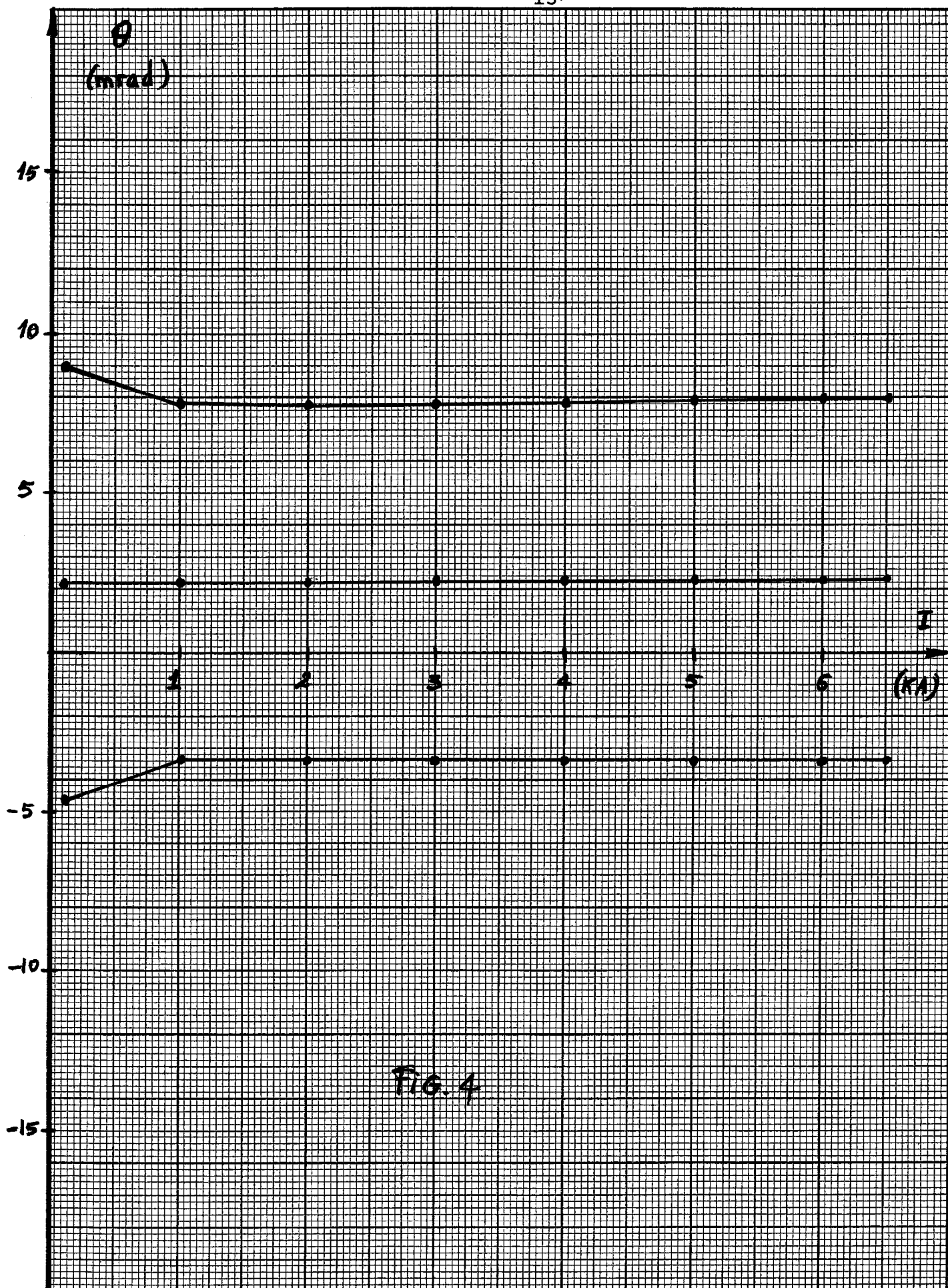


FIG. 1







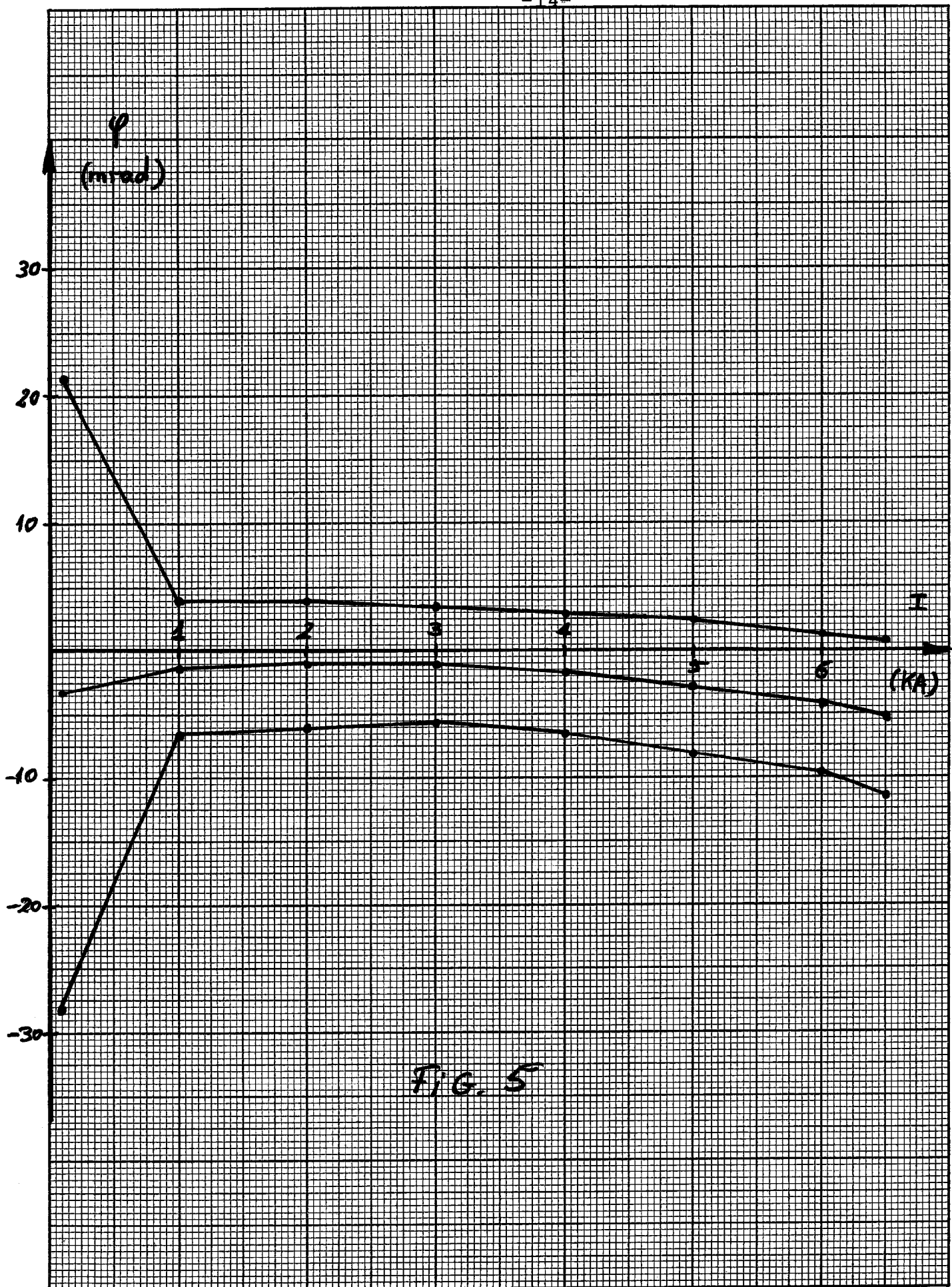


FIG. 5